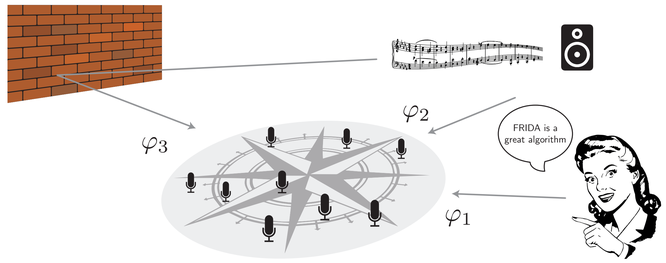
FRIDA: FRI-based DoA Estimation for Arbitrary Array Layout

Hanjie Pan, Robin Scheibler, Eric Bezzam, Ivan Dokmanić, and Martin Vetterli
Audiovisual Communications Laboratory, École Polytechnique Fédérale de Lausanne, Switzerland

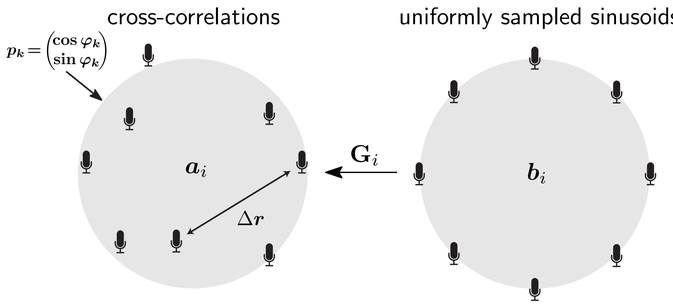
Summary

We present **FRIDA** — an algorithm for estimating directions of arrival of multiple **wideband** sound sources. The **grid-less** algorithm is applicable to **arbitrary array layouts**.

A Wishlist for DoA



Signal Model



- **Point sources** in the far field
- Cross-correlation between two microphones:

$$a_i = \sum_{k=1}^{K} \sigma_k^2(\omega_i) \mathrm{e}^{-j\omega_i \langle p_k, \Delta r\rangle}$$

- A linear mapping from the (unknown) **uniformly sampled sinusoids**

$$b_i = \sum_{k=1}^{K} \sigma_k^2(\omega_i) \underbrace{\mathrm{e}^{-jm\varphi_k}}_{\text{same for } \textit{all} \text{ subbands}}$$

to the cross-correlations a_i.

- **Annihilation equations**
  - there exists a discrete filter $h \in \mathbb{C}^{K+1}$, such that

$$b_i * h = 0 \quad \text{for } \textbf{all} \text{ subbands } i.$$

  - DoA φ_k given by the *roots* of a polynomial with coefficients h

Algorithm

- Constrained optimization

$$\min_{\substack{b_1,\cdots,b_J \\ h\in\mathcal{H}}} \quad \sum_{i=1}^{J} \|a_i - \mathbf{G}_i b_i\|_2^2$$
$$\text{subject to} \quad b_i * h = 0 \quad \text{for } i = 1, \cdots, J$$

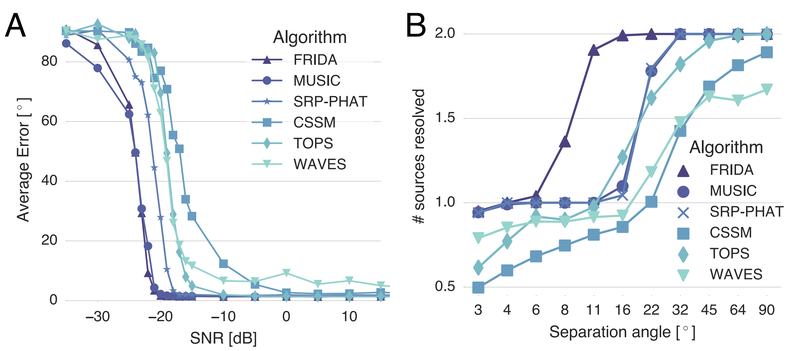
  - *non-convex* optimization
  - an **equivalent formulation** involving h only

$$\min_{h\in\mathcal{H}} \quad h^{\mathrm{H}} \Lambda(h) h$$

  - an iterative strategy [1]:

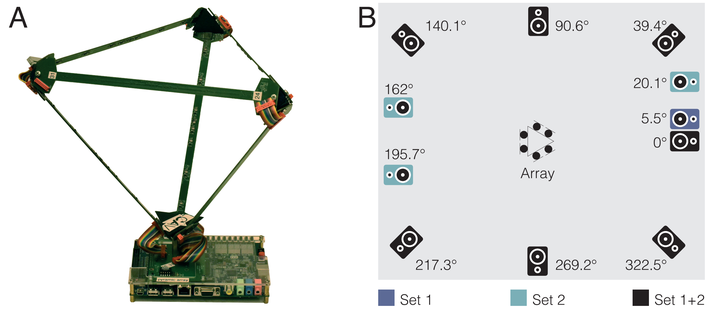
for *loop* $\leftarrow 1$ **to** *max. initializations* **do**
  Initialize h with a random vector $h^{(0)}$;
  for $n \leftarrow 1$ **to** *max. iterations* **do**
    Build $\Lambda(h)$ with $h = h^{(n-1)}$;
    Re-synthesize $b_i^{(n)}$ with the updated $h = h^{(n)}$;
    if $\sum_{i=1}^{J} \|a_i - \mathbf{G}_i b_i^{(n)}\|_2^2 \leq \varepsilon^2$ **then**
      Terminate both loops;
  $b_i \leftarrow b_i^{(n)}$, $h \leftarrow h^{(n)}$.

Simulations



A) DoA estimation errors under different **noise levels**
B) Number of resolved sources with different **separation angles**

Experimental Setup

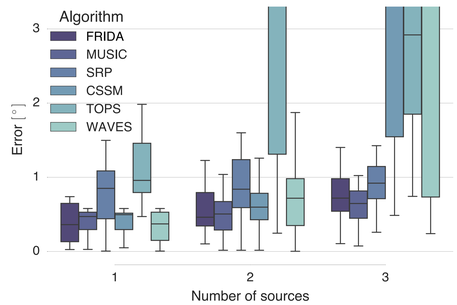


A) Pyramic array — 48 MEMS microphones and FPGA
B) Setups of the loudspeakers in the audio room

Results

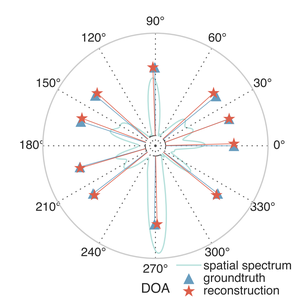
Experiment I

- Up to 3 simultaneous speech sources
- All combinations of loudspeakers (Set 1)



Experiment II

- 10 sources of white noise
- 9 microphones



Experiment III

- Two closely located sources ($5.5°$ apart)

DoA	FRIDA	MUSIC	SRP-PHAT
$0°$	$-0.5 \pm 0.4°$	$1.6 \pm 0.3°$	$1.4 \pm 0.2°$
$5.5°$	$4.6 \pm 0.2°$	$-93.9 \pm 41.2°$	$-38.1 \pm 8.6°$

Reference and Code

[1] H. Pan, T. Blu and M. Vetterli. Towards Generalized FRI Sampling with an Application to Source Resolution in Radioastronomy, in IEEE Transactions on Signal Processing, vol. 65, num. 4, p. 821-835, 2017.

Code: `http://go.epfl.ch/FRIDA`

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