Refinement of Direction of Arrival Estimators by Majorization-Minimization Optimization on the Array Manifold

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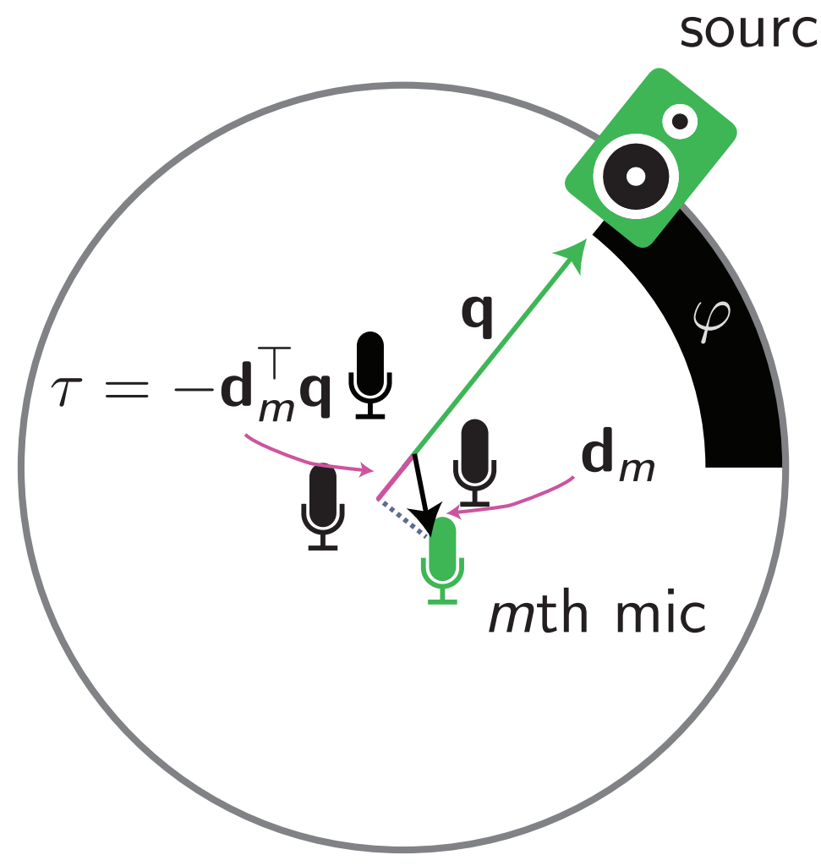
LINE

Direction of Arrival Estimators

Abstract —The key idea of this work is to refine DOA estimates by **local optimization** using **majorization-minimization**. We derive **two surrogate functions**, quadratic and linear, and validate via experiments on synthetic and recorded signals. We demonstrate up to $17\times$ **speed-up**.

Propagation Model



The measurement vector $\mathbf{x}_{kn} \in \mathbb{C}^M$ is

$$\mathbf{x}_{kn} = \mathbf{a}_k(\mathbf{q}) y_{kn} + \text{noise}$$

with direction vector $\mathbf{q} \in \mathbb{R}^3$, $\|\mathbf{q}\| = 1$, and with the **steering vectors**

$$\mathbf{a}_k(\mathbf{q}) = \left[\cdots\ e^{j\omega_k \mathbf{d}_m^\top \mathbf{q}}\ \ldots\right]^\top$$

Generalized DOA Formulation

Goal Find local minima/maxima of

$$\mathcal{J}(\mathbf{q}) = \sum_{k=1}^{K} \mathbf{a}_k(\mathbf{q})^H \mathbf{V}_k \mathbf{a}_k(\mathbf{q}), \quad \text{s.t.} \quad \begin{cases} \|\mathbf{q}\| = 1 \\ \mathbf{V}_k \succeq 0 \text{ (PSD)} \end{cases}$$

Method	Opt	$\mathbf{V}_k$
SRP	Max	$\mathbb{E}[\mathbf{x}_{fn}\mathbf{x}_{fn}^H]$
MUSIC	Min	$\mathbb{E}[\mathbf{n}_{fn}\mathbf{n}_{fn}^H]$ (cov. mat. noise)
MVDR	Min	$\mathbb{E}[\mathbf{x}_{fn}\mathbf{x}_{fn}^H]^{-1}$

Objective is a Sum of Cosine

$$\mathcal{J}(\mathbf{q}) = 2\sum_{n>m} u_{mn} \cos(\psi_{mn} - \omega_k \Delta_{mn}^\top \mathbf{q}) + \text{const.}$$

with $\Delta_{mn} = \mathbf{d}_m - \mathbf{d}_n$, $u_{mn} = |(\mathbf{V}_k)_{mn}|$, $\psi_{mn} = \arg((\mathbf{V}_k)_{mn})$.

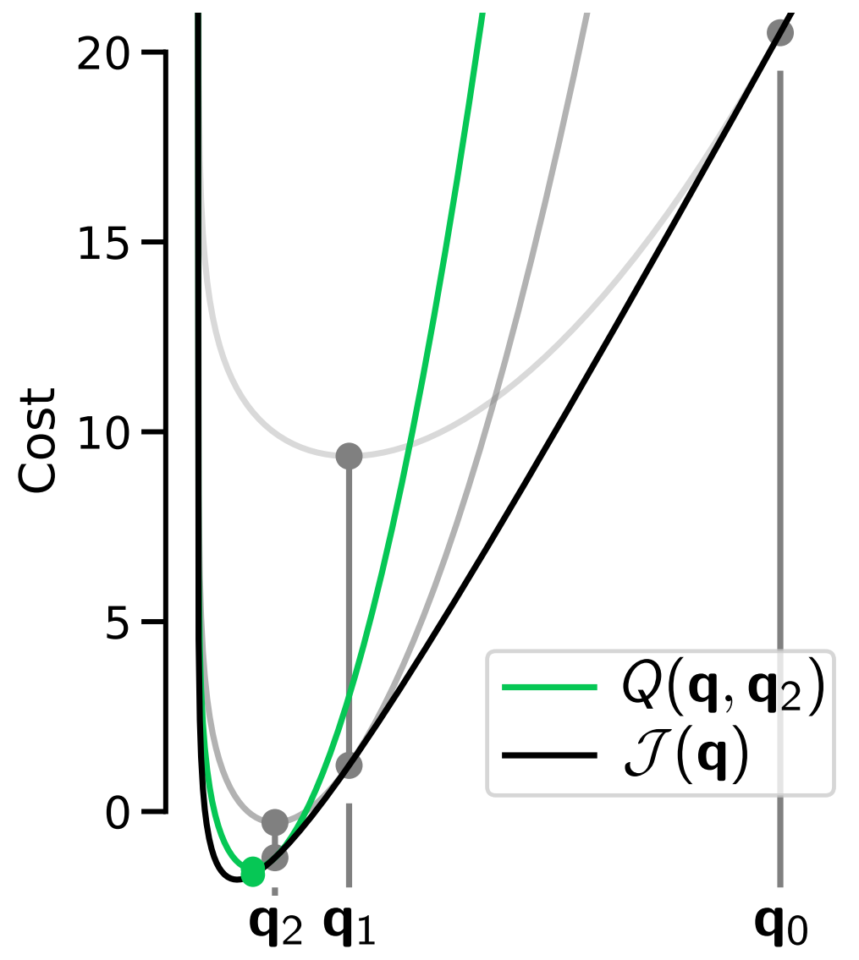
Conventional Optimization: Grid Search

1. Sample search space at locations $\hat{\mathbf{q}}_1, \ldots, \hat{\mathbf{q}}_L$
2. Choose $\mathbf{q}^\star = \arg\min_{\ell \in \{1,\ldots,L\}} \mathcal{J}(\hat{\mathbf{q}}_\ell)$

Problems

- Precision depends on L
- Curse of dimensionality ($L > 10^4$ for ~ 2 error in 3D)

Refinement by MM Optimization

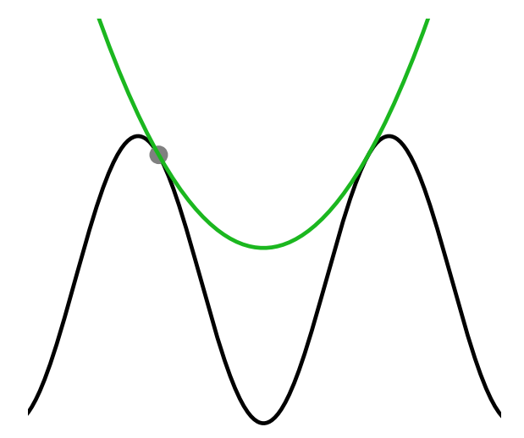


1. Find initial DOA estimate with rough grid
2. Refine with MM iterations

$$\mathbf{q}_t \leftarrow \arg\min_{\mathbf{q},\, \|\mathbf{q}\|=1} Q(\mathbf{q}, \mathbf{q}_{t-1})$$

where $Q(\mathbf{q}, \hat{\mathbf{q}})$ is a **surrogate** of $\mathcal{J}(\mathbf{q})$.

Key Ingredient: Quadratic Surrogate of Cosine [1]



Let $\theta, \theta_0 \in \mathbb{R}$, $z_0 = \arg\min_{z\in\mathbb{Z}} |\theta_0 + 2\pi z|$, and $\phi_0 = \theta_0 + 2\pi z_0$. Then,

$$-\cos(\theta) \leq \frac{1}{2}\operatorname{sinc}(\phi_0)(\theta + 2\pi z_0)^2 + \ldots$$

Quadratic Surrogate

The previous inequality is directly applicable to the objective

$$\mathcal{J}(\mathbf{q}) = \sum_{m>n} u_{mn} \cos(\psi_{mn} - \Delta_{mn}^\top \mathbf{q}) \leq \sum_{mn} \hat{u}_{mn}(\hat{\psi}_{mn} - \Delta_{mn}^\top \mathbf{q})^2 + \ldots$$

where $\hat{u}_{mn}$ and $\hat{\psi}_{mn}$ depend on $\mathbf{q}_{t-1}$. This gives the update

$$\mathbf{q}_t \leftarrow \arg\min_{\mathbf{q}\in\mathbb{R}^3} \mathbf{q}^\top \mathbf{D}(\mathbf{q}_{t-1})\mathbf{q} - 2\mathbf{v}(\mathbf{q}_{t-1})^\top \mathbf{q} \quad \text{subject to} \quad \|\mathbf{q}\|^2 = 1 \quad (1)$$

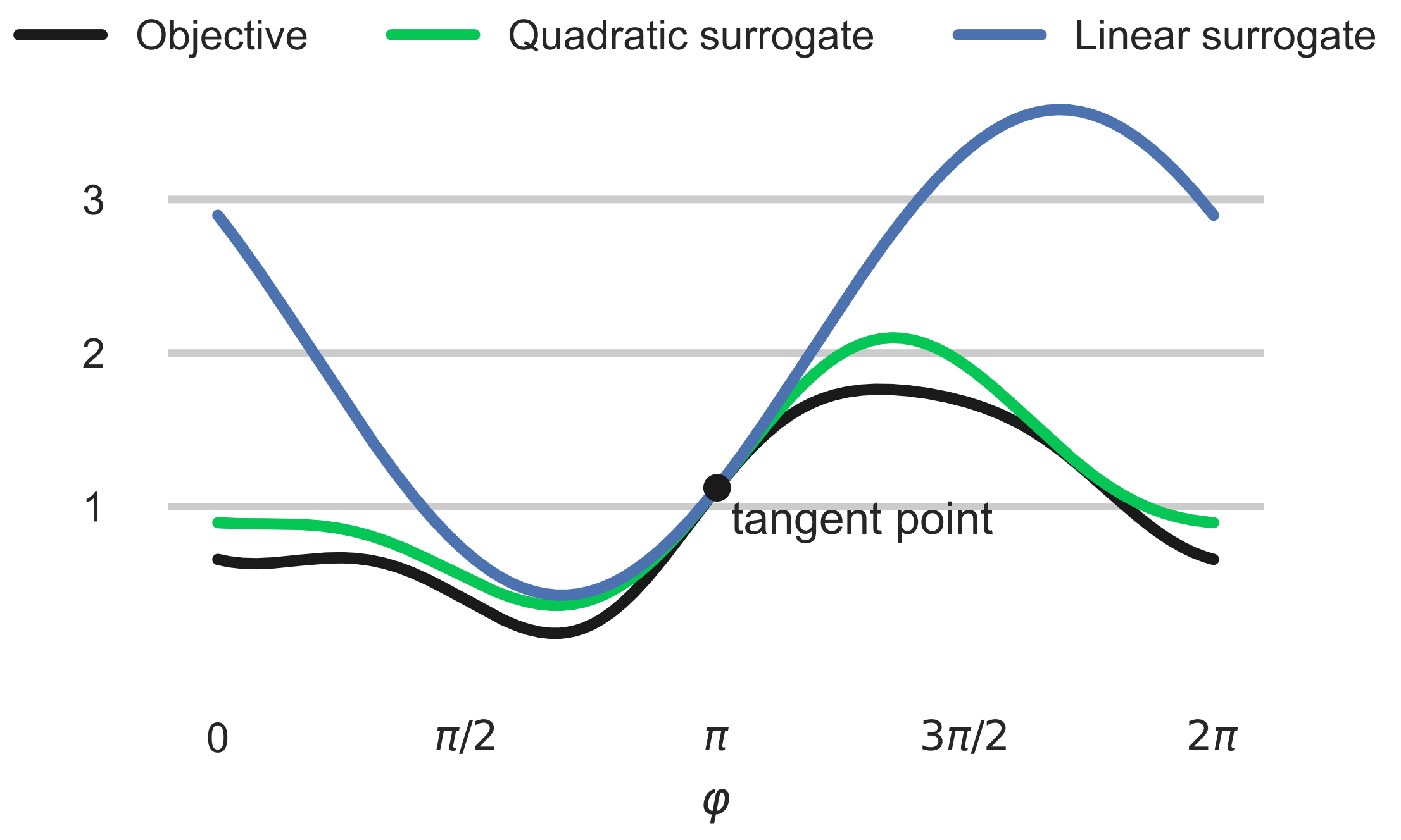
with $\mathbf{D}(\mathbf{q}_{t-1}) = \sum_{mn} \hat{u}_{mn}\Delta_{mn}\Delta_{mn}^\top$, and $\mathbf{v}(\mathbf{q}_{t-1}) = \sum_{mn} \hat{u}_{mn}\hat{\psi}_{mn}\Delta_{mn}$. Efficient algorithm to solve (1) is available [2]

Linear Surrogate

A quadratic on bounded domain admits a linear surrogate:

$$\mathbf{q}_t \leftarrow \arg\min_{\mathbf{q}\in\mathbb{R}^3, \|\mathbf{q}\|^2=1} -\left(\mathbf{v}(\mathbf{q}_{t-1}) - \mathbf{D}(\mathbf{q}_{t-1})\mathbf{q}_{t-1} + C(\mathbf{q}_{t-1})\mathbf{q}_{t-1}\right)^\top \mathbf{q}$$

which has a closed-form solution.



Experimental Validation

- Baseline: grid-search with 10000 points
- Proposed: grid-search with 100 points + 30 iterations MM

Synthetic Reverberant Speech

Median Error, 12 channels, reverb. time ≈ 500 ms, 100 rep.

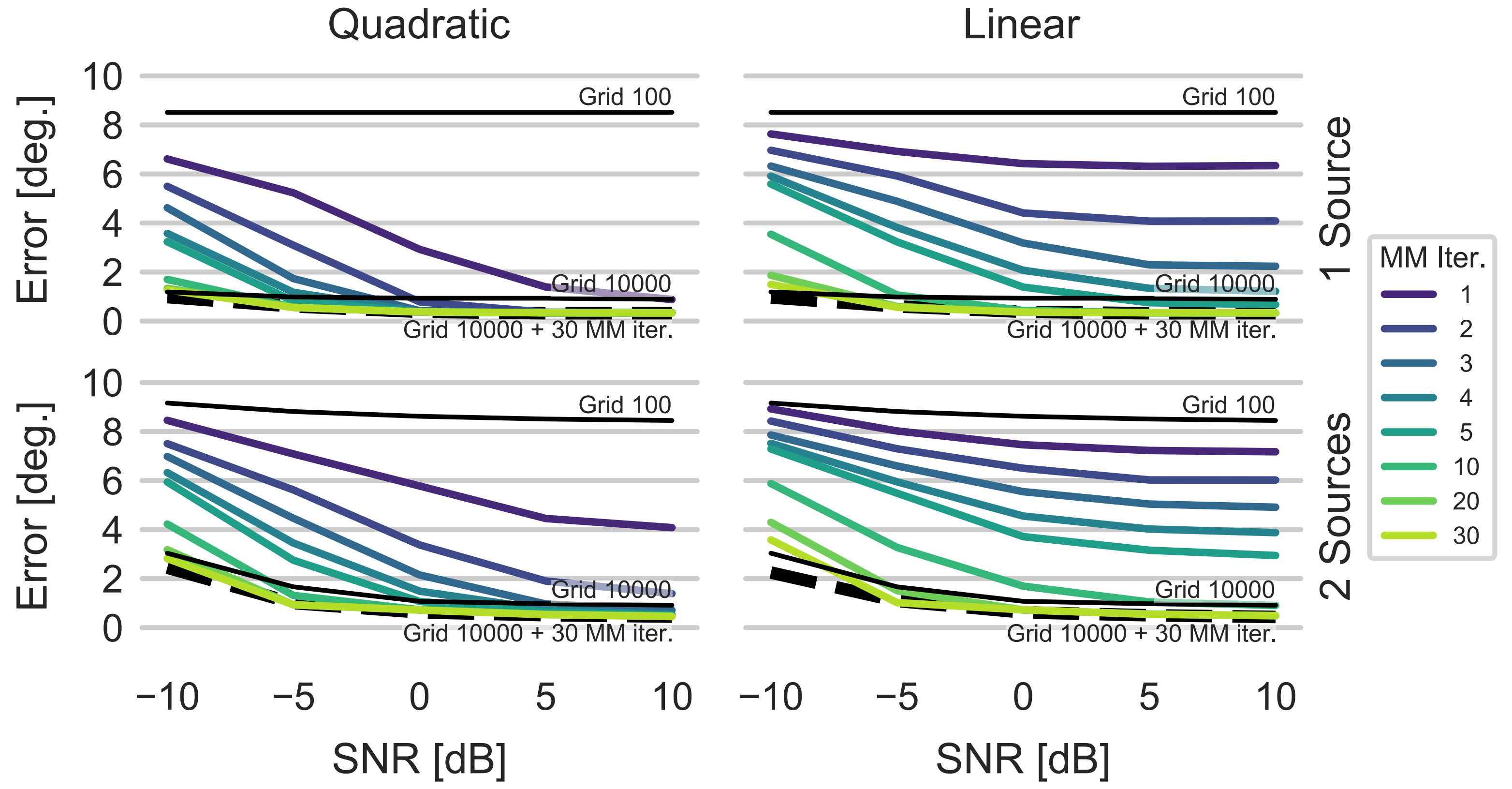
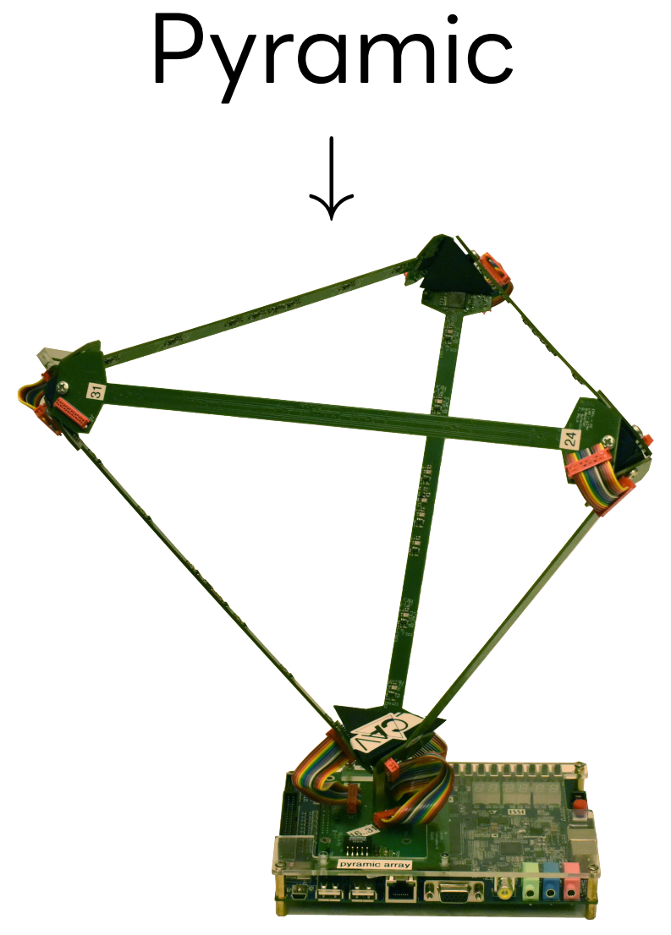


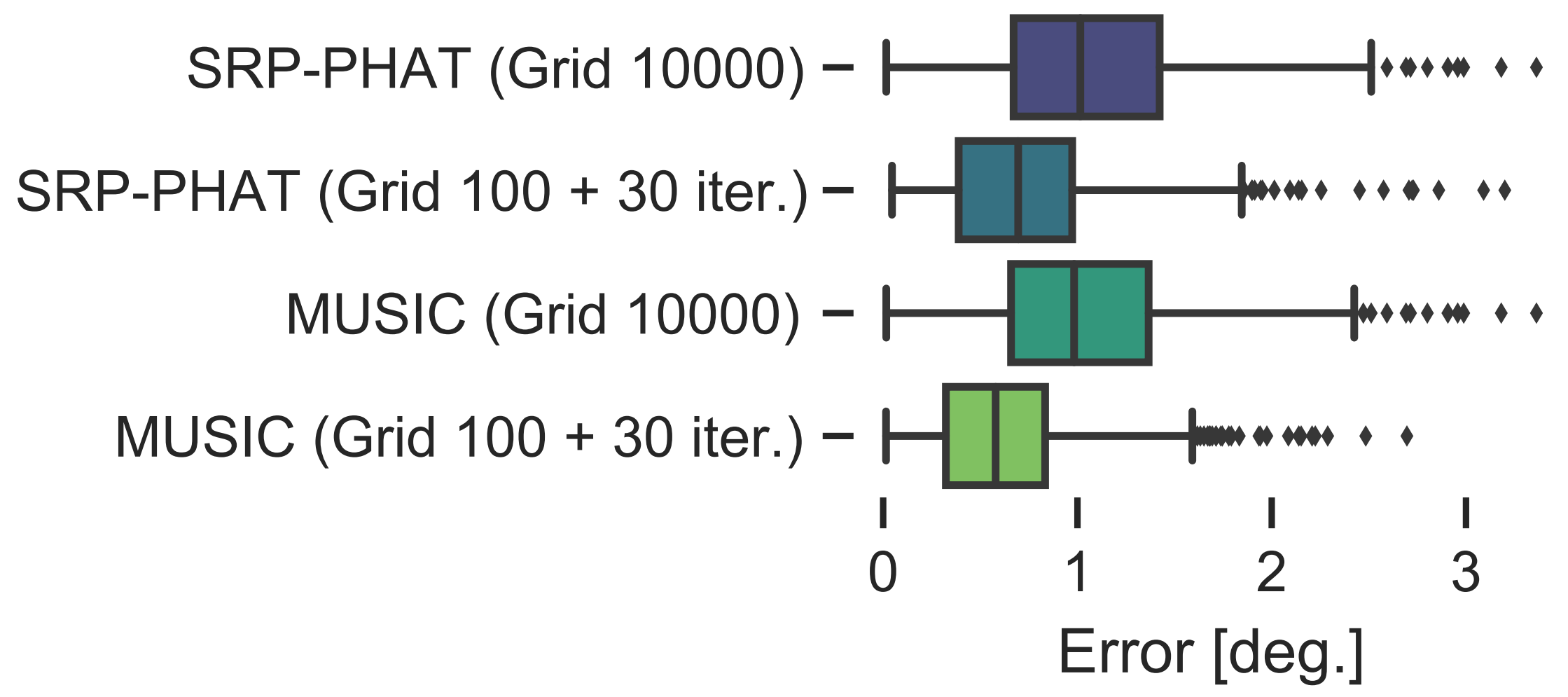
Table 1: Median runtimes in seconds with the quadratic surrogate.

Description	Grid	MM Iter.	SRP-PHAT 1 src	SRP-PHAT 2 src	MUSIC 1 src	MUSIC 2 src
fine grid-search	10000	0	4.55	4.58	4.57	4.48
proposed method	100	30	0.35	0.42	0.27	0.37
speed-up			13×	11×	17×	12×

Recorded Anechoic Speech

Pyramic 48-channel array, anechoic, 540 positions [3]





References

[1] K. Yamaoka et al., Proc. WASPAA, Oct. 2019, pp. 130–134.
[2] J. J. More, Optim. Method Softw., vol. 2, no. 3–4, pp. 189–209, Jan. 1993.
[3] R. Scheibler et al., Proc. IWAENC, Sep. 2018, pp. 226–230.
[4] `https://github.com/LCAV/pyroomacoustics`
[5] `https://github.com/fakufaku/doamm`