Independent Vector Analysis via Log-quadratically Penalized Quadratic Minimization

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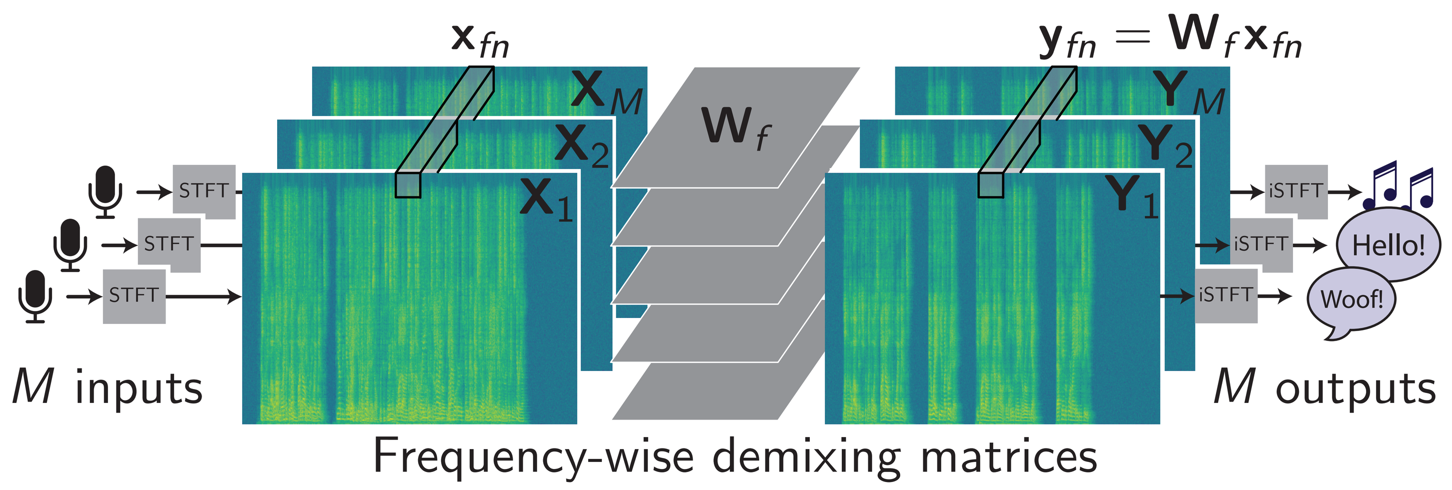
Faster Blind Source Separation

Abstract —We propose a new algorithm for AuxIVA based BSS using **majorization-minimization**. Along the way, we solve a new type of non-convex optimization problem that we call **log-quadratically penalized quadratic minimization**.

Blind Source Separation by Independent Vector Analysis



Likelihood Function of Observed Data

$$\mathcal{L}(\{\mathbf{W}_f\} \mid \underbrace{\mathbf{X}_1, \ldots, \mathbf{X}_M}_{\text{observation}}) = \underbrace{\prod_{m=1}^{M} p(\mathbf{Y}_m)}_{\text{independence}} \underbrace{\prod_{f=1}^{F} |\det(\mathbf{W}_f)|^{2N}}_{\text{change of variable}}$$

Independent Vector Analysis [1, 2]

Estimate $\mathbf{W}_f$ by minimizing log-likelihood ($G(\mathbf{Y}) = -\log p(\mathbf{Y})$)

$$\ell(\{\mathbf{W}_f\}) \approx \sum_m G(\mathbf{Y}_m) - 2N \sum_f \log|\det \mathbf{W}_f|$$

AuxIVA [3]: Majorization-Minimization of $\ell(\{\mathbf{W}_f\})$

Hypothesis We can majorize the log-pdf of the source

$$G(\mathbf{Y}) \leq \sum_{fn} \widehat{G}_{fn}(\mathbf{Y}) |(\mathbf{Y})_{fn}|^2$$

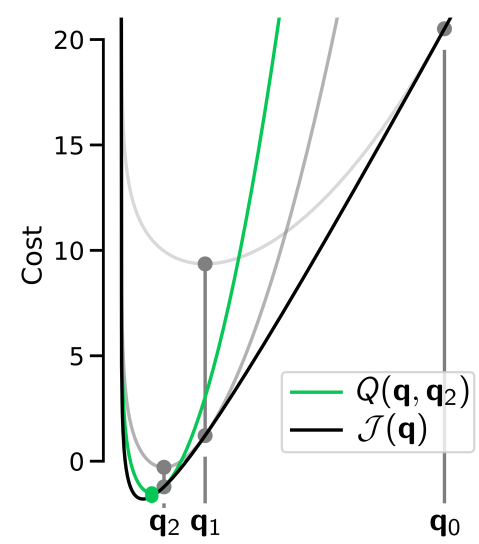
Then there exists the **upper bound** function

$$\ell(\{\mathbf{W}_f\}) \lesssim \ell_+(\{\mathbf{W}_f\}) = \sum_f \left[\sum_m \mathbf{w}_{mf}^H \mathbf{V}_{mf} \mathbf{w}_{mf} - 2\log|\det \mathbf{W}_f| \right]$$

Ideal AuxIVA Algorithm

for loop $\leftarrow 1$ **to** max. iterations **do**

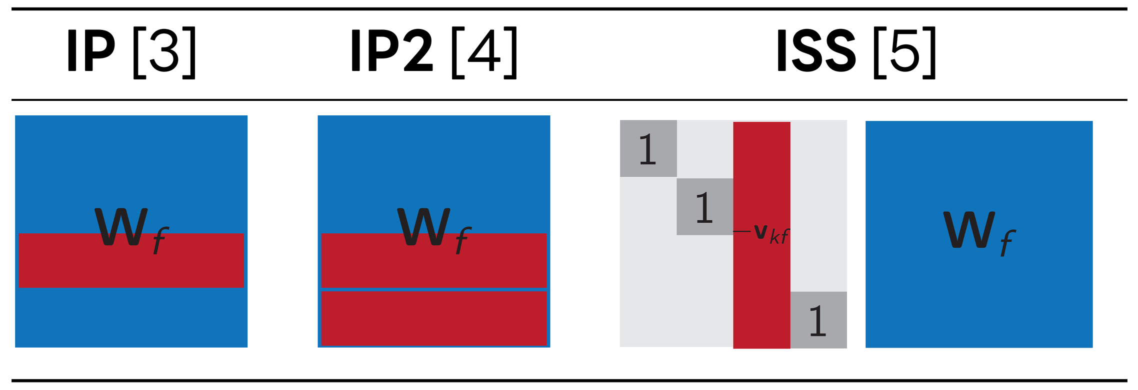
- $\mathbf{Y}_m \leftarrow \text{demix}(\{\mathbf{W}_f\}, \mathbf{X}_1, \ldots, \mathbf{X}_M)$
- $\mathbf{V}_{mf} = \frac{1}{N} \sum_n \widehat{G}_{fn}(\mathbf{Y}_m) \mathbf{x}_{fn} \mathbf{x}_{fn}^H$
- $\mathbf{W}_f \leftarrow \underset{\mathbf{W} \in \mathbb{C}^{M \times M}}{\arg\min} \sum_m \mathbf{w}_m^H \mathbf{V}_{mf} \mathbf{w}_m - 2\log|\det \mathbf{W}|$



Problem No closed form solution for the minimization!

Block Coordinate Descent Algorithm

Minimize wrt to only part of $\mathbf{W}_f$



Iterative Projection Adjustement

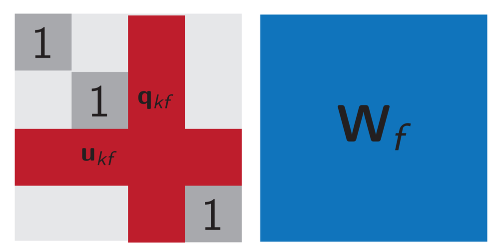
Multiplicative updates of $\mathbf{W}_f$ by

$$\mathbf{T}_m(\mathbf{u}, \mathbf{q}) = \mathbf{I} + \mathbf{e}_m(\mathbf{u} - \mathbf{e}_m)^H + \mathbf{q}\mathbf{e}_m^\top$$

Apply M updates to $\mathbf{W}_f$ sequentially

for loop $\leftarrow 1$ **to** M **do**

- $\mathbf{u}_m, \mathbf{q}_m \leftarrow \underset{\mathbf{u},\mathbf{q} \in \mathbb{C}^M}{\arg\min}\ \ell_+(\mathbf{T}_m(\mathbf{u}, \mathbf{q})\mathbf{W}_f)$
- $\mathbf{W}_f \leftarrow \mathbf{T}_m(\mathbf{u}_m, \mathbf{q}_m)\mathbf{W}_f$



Solving the Update Equation

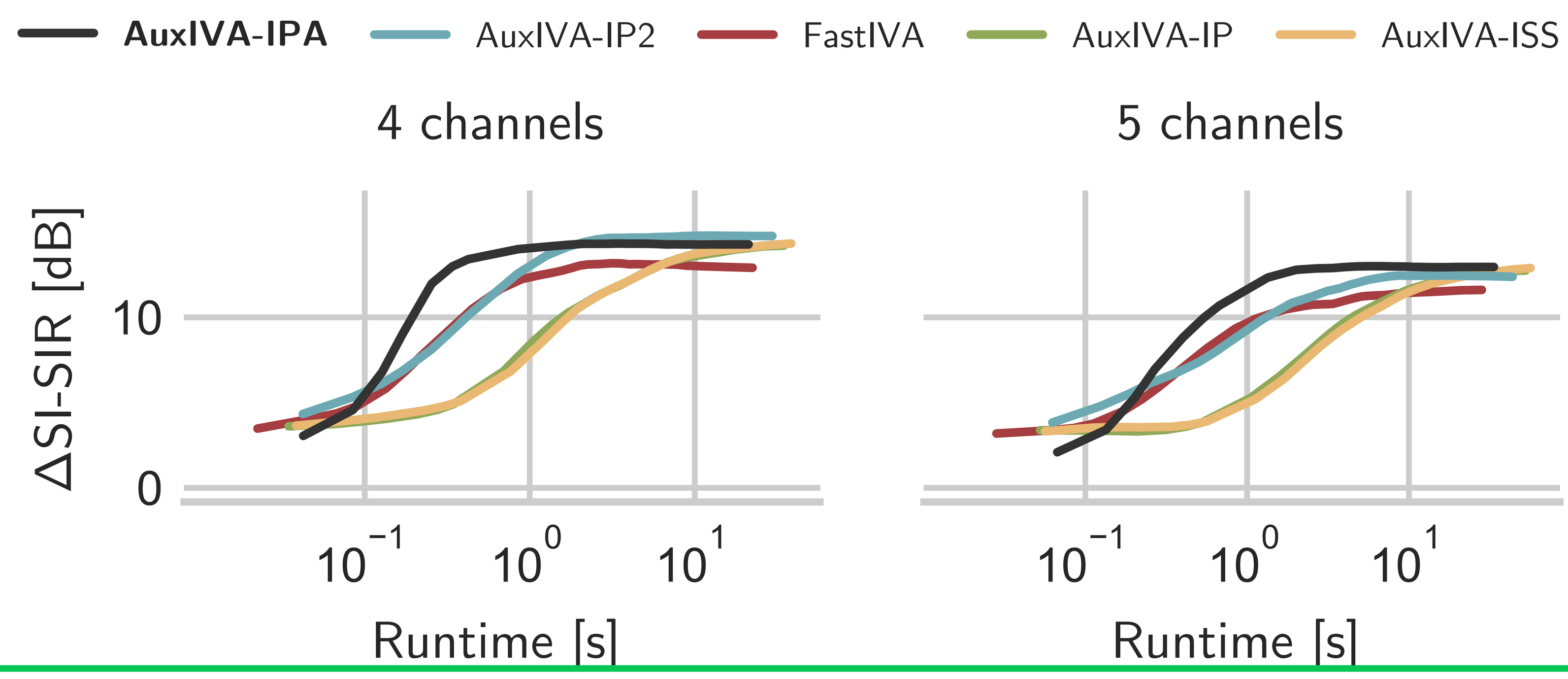
1. For $\mathbf{u}$, closed-form as a function of $\mathbf{q}$ exists
2. Replace $\mathbf{u}^*(\mathbf{q})$ in the objective leads to new problem
3. Solve **Log-Quadratically Penalized Quadratic Minimization**

$$\min_{\mathbf{q} \in \mathbb{C}^d} \mathbf{q}^H \mathbf{q} - \log((\mathbf{q} + \mathbf{v})^H \mathbf{U} (\mathbf{q} + \mathbf{v}) + z) \qquad \text{(LQPQM)}$$

where $\mathbf{U} \in \mathbb{C}^{d \times d}$ PSD, $\mathbf{v} \in \mathbb{C}^d$, $z \geq 0$.

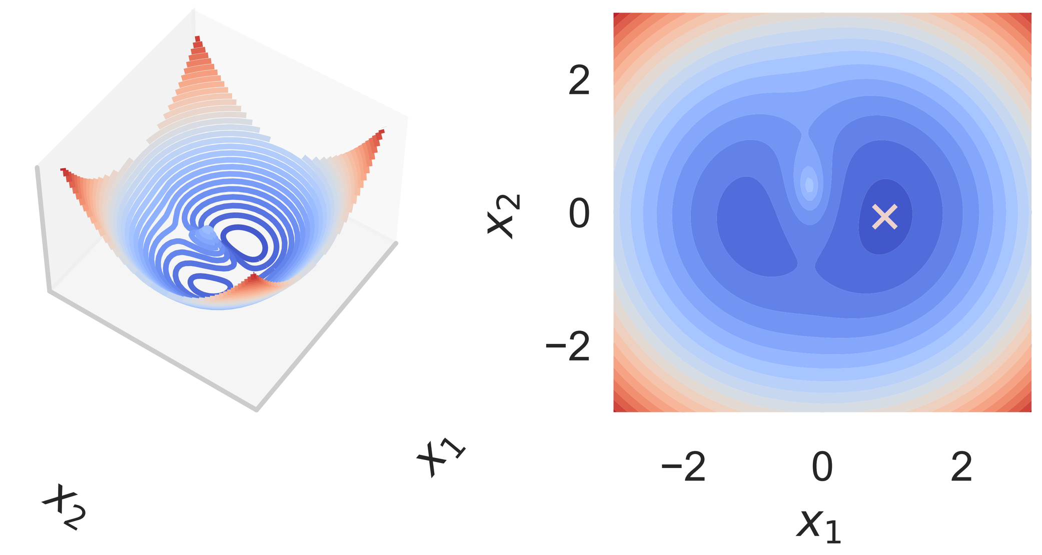
Experiments

Convergence (SI-SIR) as function of runtime
(16 kHz, 1000 sim. rooms, SNR 15 dB)



Solving LQPQM

The loss landscape of LQPQM has several local optima



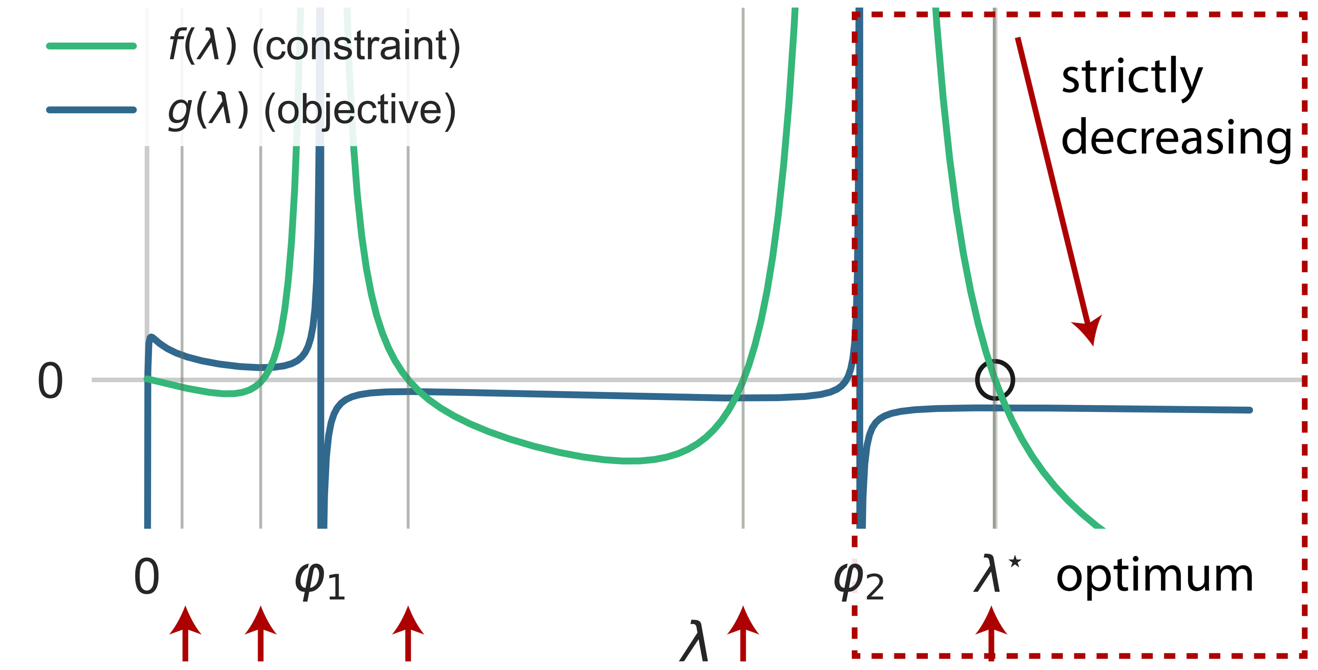
Solving LQPQM

We can reduce the LQPQM to a 1D problem

$$\min_{\lambda \in \mathbb{R}_+} g(\lambda) \quad \text{subject to} \quad f(\lambda) = 0$$

where

$$f(\lambda) = \lambda^2 \sum_{m=1}^{d} \frac{\varphi_m |\hat{v}_m|^2}{(\lambda - \varphi_m)^2} + z - \lambda = 0.$$



Theorem

- $g(\lambda_1) \leq g(\lambda_2)$ if $f(\lambda_1) = f(\lambda_2) = 0$ and $\lambda_1 < \lambda_2$
- For $\lambda > \phi_{\max}$
  - One, and only one, zero
  - $f(\lambda)$ strictly descreasing

Thus λ^* is the **global minimum!**

References

[1] Kim et al., Proc. ICA, 2006.
[2] Hiroe, Proc. ICA, 2006.
[3] Ono, Proc. WASPAA, 2011.
[4] Ono, Proc. ASJ, 2018.
[5] Scheibler, Ono, Proc. ICASSP, 2020.