Independent Vector Analysis via Log-quadratically Penalized Quadratic Minimization

Robin Scheibler

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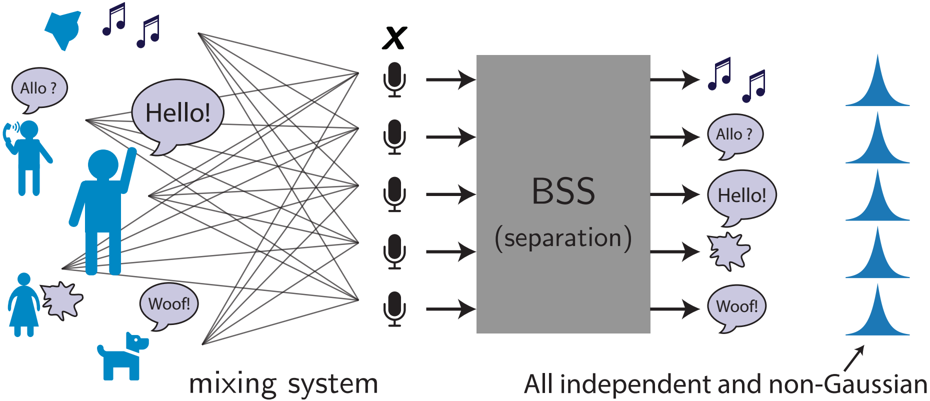
LINE

Outline

1. AuxIVA: BSS with Majorization-Minimization
2. Iterative Projection Adjustment
3. Log-quadratically Penalized Quadratic Minimization
4. Experiment: Speed Contest

AuxIVA: BSS with Majorization-Minimization

Blind Source Separation



Contributions

1. New algorithm for independent vector analysis
2. Solution to new non-convex problem

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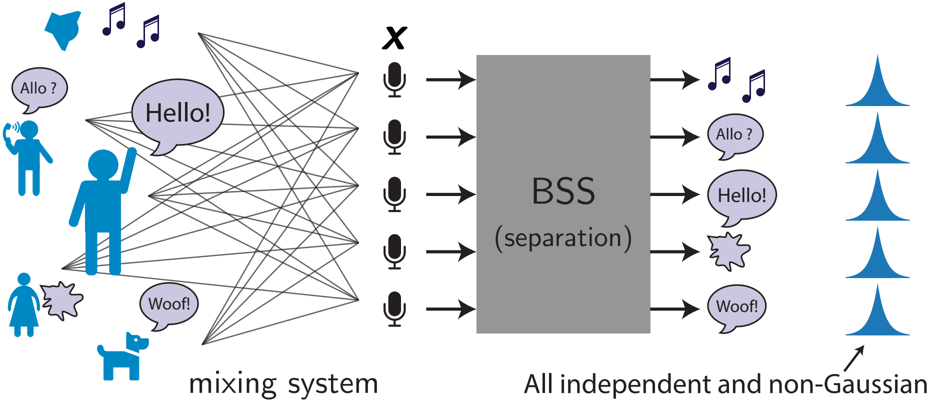

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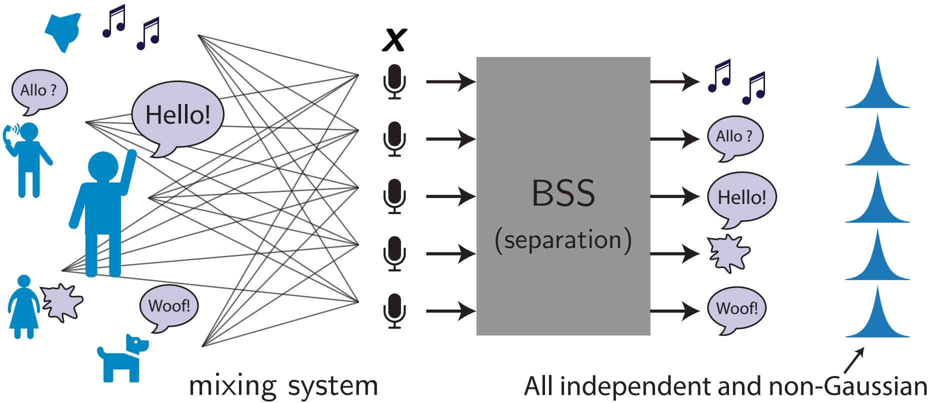

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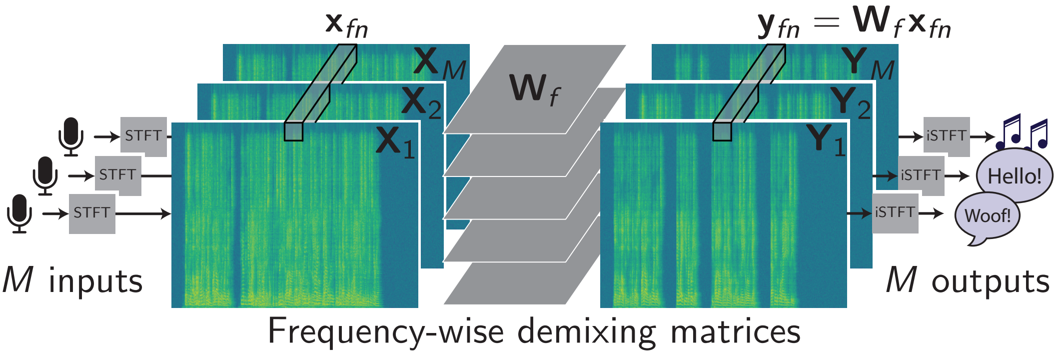
Blind Source Separation by Independent Vector Analysis



Likelihood Function of Observed Data

$$\mathcal{L}(\{\mathbf{W}_f\} \mid \underbrace{\mathbf{X}_1, \ldots, \mathbf{X}_M}_{\text{observation}}) = \underbrace{\prod_{m=1}^{M} p(\mathbf{Y}_m)}_{\text{independence}} \underbrace{\prod_{f=1}^{F} |\det(\mathbf{W}_f)|^{2N}}_{\text{change of variable}}$$

Maximum Likelihood Estimation

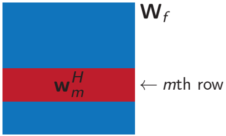
Estimate $\mathbf{W}_f$ by minimizing log-likelihood function ($G(\mathbf{Y}) = -\log p(\mathbf{Y})$)

$$\ell(\{\mathbf{W}_f\}) = -\log \mathcal{L}(\{\mathbf{W}_f\} \mid \mathbf{X}_1, \ldots, \mathbf{X}_M) \approx \sum_m G(\mathbf{Y}_m) - 2N \sum_f \log |\det \mathbf{W}_f|$$

AuxIVA [Ono2011]: Majorization-Minimization of $\ell(\{\mathbf{W}_f\})$

Hypothesis We can majorize the log-pdf of the source

$$G(\mathbf{Y}) \leq \sum_{fn} \widehat{G}_{fn}(\mathbf{Y}) |(\mathbf{Y})_{fn}|^2$$



Then there exists the **upper bound** function

$$\ell(\{\mathbf{W}_f\}) \lesssim \ell_+(\{\mathbf{W}_f\}) = \sum_f \left[\sum_m \mathbf{w}_{mf}{}^H \mathbf{V}_{mf} \mathbf{w}_{mf} - 2 \log |\det \mathbf{W}_f| \right]$$

Majorization-Minimization Optimization

We want to solve

$$\min_{\theta} f(\theta)$$

Let **surrogate func.** $Q(\theta, \hat{\theta})$ be

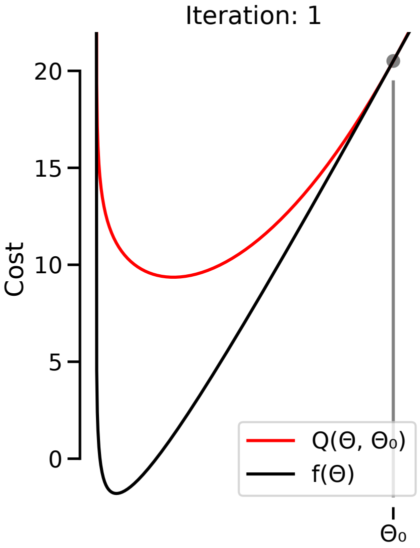
1. $Q(\theta, \hat{\theta}) \geq f(\theta)$
2. $Q(\hat{\theta}, \hat{\theta}) = f(\hat{\theta})$

The sequence $t = 0, \ldots, T$,

$$\theta_{t+1} \leftarrow \arg\min_{\theta} Q(\theta, \theta_t)$$

guarantees

$$f(\theta_0) \geq \ldots \geq f(\theta_T)$$



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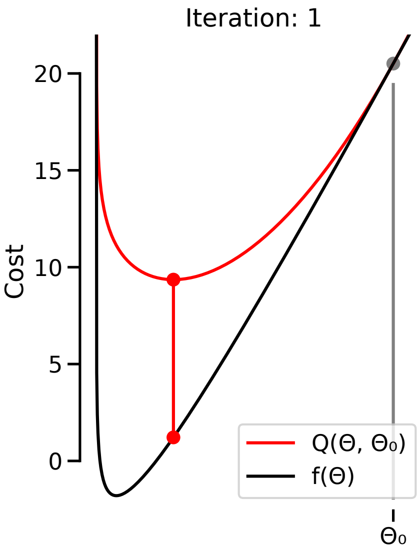

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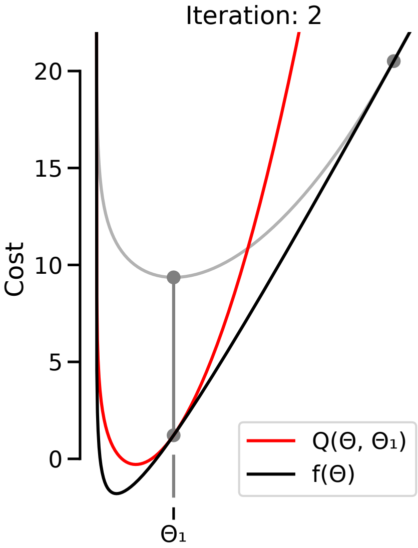

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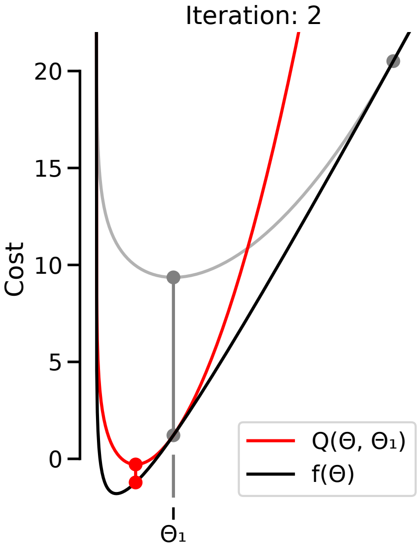

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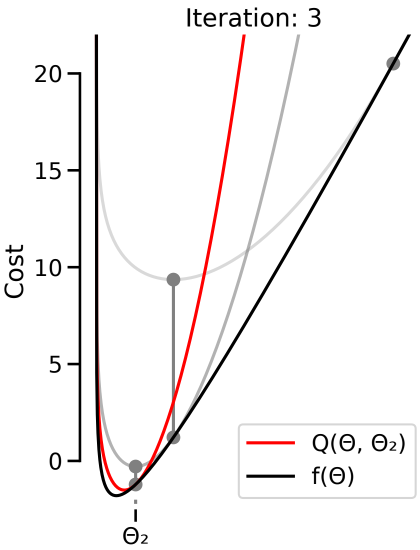

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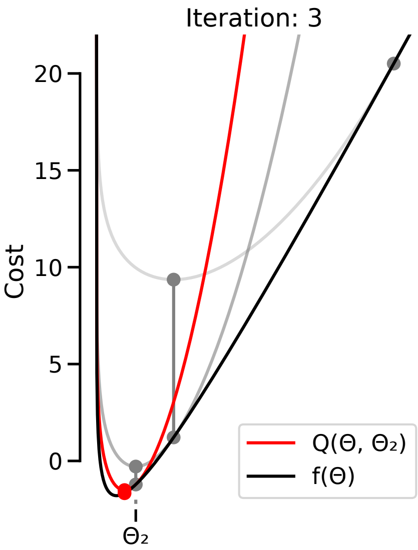

Ideal AuxIVA Algorithm

Initialize $\mathbf{W}_f$ (often $\mathbf{I}$)

for loop $\leftarrow$ 1 **to** max. iterations **do**

$$\mathbf{Y}_m \leftarrow \text{demix}(\{\mathbf{W}_f\}, \mathbf{X}_1, \ldots, \mathbf{X}_M)$$

$$\mathbf{V}_{mf} = \frac{1}{N} \sum_n \widehat{G}_{fn}(\mathbf{Y}_m)\mathbf{x}_{fn}\mathbf{x}_{fn}^H$$

$$\mathbf{W}_f \leftarrow \underset{\mathbf{W} \in \mathbb{C}^{M \times M}}{\arg\min} \sum_m \mathbf{w}_m{}^H \mathbf{V}_{mf} \mathbf{w}_m - 2 \log |\det \mathbf{W}|$$

Problem No closed-form solution to the last step, aka HEAD [Yeredor2009]

$$\min_{\mathbf{W} \in \mathbb{C}^{M \times M}} \sum_m \mathbf{w}_m{}^H \mathbf{V}_{mf} \mathbf{w}_m - 2 \log |\det \mathbf{W}| \qquad \text{(HEAD)}$$

Solution Solve for part of $\mathbf{W}$ only (i.e., block coordinate descent)

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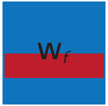
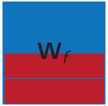
Block Coordinate Descent Algorithms

Iterative Projection (IP) [Ono2011]



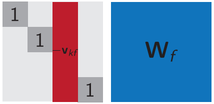
- The original AuxIVA algorithm
- Updates a single row of $\mathbf{W}_f$ at a time

Iterative Projection 2 (IP2) [Ono2018]



- Updates a two rows of $\mathbf{W}_f$ at a time
- Faster convergence

Iterative Source Steering (ISS) [Scheibler2020]



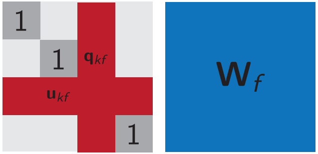
- Updates one steering vector at a time
- Low complexity algorithm

Iterative Projection Adjustment

Proposed Method: Iterative Projection Adjustment (IPA)

Multiplicative updates of $\mathbf{W}_f$ by

$$\mathbf{T}_m(\mathbf{u}, \mathbf{q}) = (\mathbf{I} + \mathbf{e}_m(\mathbf{u} - \mathbf{e}_m)^H + \mathbf{q}\mathbf{e}_m^\top)$$



Apply M updates to $\mathbf{W}_f$ sequentially

for loop $\leftarrow$ 1 **to** M **do**

$$\mathbf{u}_m, \mathbf{q}_m \leftarrow \underset{\mathbf{u},\mathbf{q}\in\mathbb{C}^M}{\arg\min}\, \ell_+(\mathbf{T}_m(\mathbf{u}, \mathbf{q})\mathbf{W}_f)$$

$$\mathbf{W}_f \leftarrow \mathbf{T}_m(\mathbf{u}_m, \mathbf{q}_m)\mathbf{W}_f$$

Contribution: Exact Solution for Update Equation

$$\min_{\mathbf{u},\mathbf{q}\in\mathbb{C}^M} \sum_{k\neq m} (\mathbf{e}_k + q_k\mathbf{e}_m)^H \mathbf{V}_k (\mathbf{e}_k + q_k\mathbf{e}_m) + \mathbf{u}^H\mathbf{V}_m\mathbf{u} - 2\log|\det(\mathbf{I} + \mathbf{e}_m(\mathbf{u} - \mathbf{e}_m)^H + \mathbf{q}\mathbf{e}_m^\top)|$$

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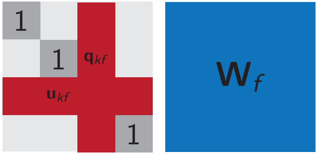

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Solving the IPA Update Equation

Sketch of Solution

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1. For $\mathbf{u}$, closed-form as a function of $\mathbf{q}$ exists
2. Replace $\mathbf{u}^\star(\mathbf{q})$ in the objective leads to new problem
3. Solve **Log-Quadratically Penalized Quadratic Minimization (LQPQM)**

$$\min_{\mathbf{q}\in\mathbb{C}^d} \mathbf{q}^H\mathbf{q} - \log((\mathbf{q}+\mathbf{v})^H\mathbf{U}(\mathbf{q}+\mathbf{v}) + z) \qquad \text{(LQPQM)}$$

where $\mathbf{U} \in \mathbb{C}^{d\times d}$ PSD, $\mathbf{v} \in \mathbb{C}^d$, $z \geq 0$.

Contribution 2

Algorithm to compute global minimum of LQPQM

Solving the IPA Update Equation

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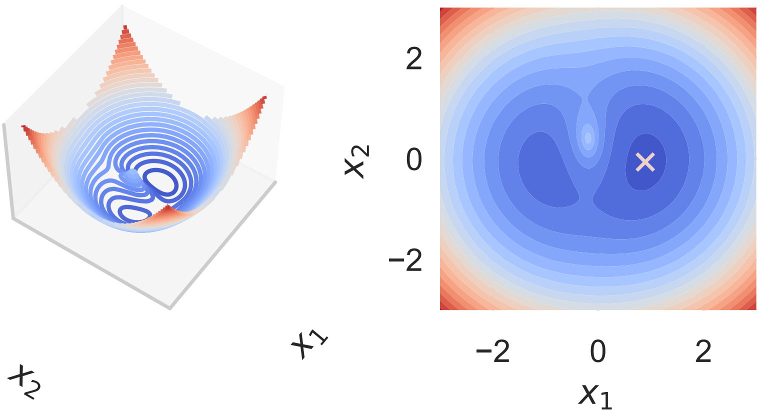
Log-quadratically Penalized Quadratic Minimization

LQPQM: Loss Landscape in 2D

$$\min_{\mathbf{q}\in\mathbb{C}^d} \mathbf{q}^H\mathbf{q} - \log((\mathbf{q}+\mathbf{v})^H\mathbf{U}(\mathbf{q}+\mathbf{v})+z), \quad \mathbf{U}\in \mathrm{PSD}, \mathbf{v}\in\mathbb{C}^d, z\geq 0. \qquad \text{(LQPQM)}$$



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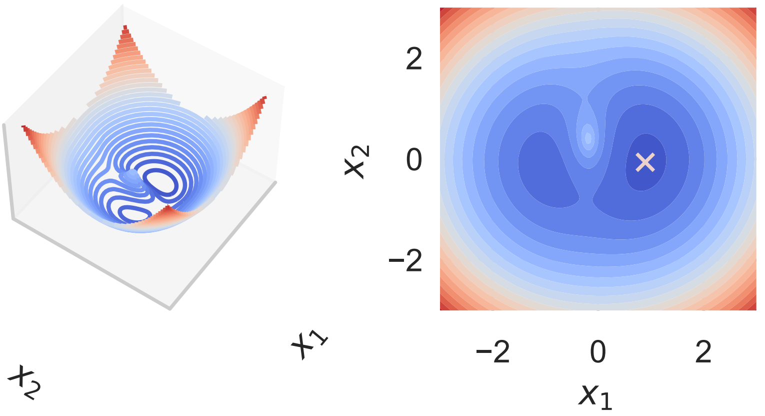

Solve LQPQM

Optimality Conditions ($\nabla \mathcal{J}(\mathbf{q}) = 0$)

$$\nabla \mathcal{J}(\mathbf{q}) = \mathbf{q} - \frac{\mathbf{U}(\mathbf{q} - \mathbf{v})}{(\mathbf{q} + \mathbf{v})^H \mathbf{U}(\mathbf{q} + \mathbf{v}) + z} = \mathbf{0} \Leftrightarrow \begin{cases} \mathbf{q} & = \frac{1}{\lambda}\mathbf{U}(\mathbf{q} + \mathbf{v}) \quad \text{(A1)} \\ \lambda & = (\mathbf{q} + \mathbf{v})^H \mathbf{U}(\mathbf{q} + \mathbf{v}) + z \quad \text{(A2)} \end{cases}$$

Reduce to function of λ only

- Solve A1: $\mathbf{q}(\lambda) = (\lambda \mathbf{I} - \mathbf{U})^{-1}\mathbf{U}\mathbf{v}$
- Replace in A2, work in eigenbasis of $\mathbf{U}$
- A2 then leads to constraint (φ_m eigenvalues of $\mathbf{U}$, $\hat{\mathbf{v}}$ is $\mathbf{v}$ in $\mathbf{U}$ eigenbasis)

$$f(\lambda) = \lambda^2 \sum_{m=1}^{d} \frac{\varphi_m |\hat{v}_m|^2}{(\lambda - \varphi_m)^2} + z - \lambda = 0.$$

- Similarly, we can express objective value as $g(\lambda)$

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- A2 then leads to constraint (φ_m eigenvalues of $\mathbf{U}$, $\hat{\mathbf{v}}$ is $\mathbf{v}$ in $\mathbf{U}$ eigenbasis)

$$f(\lambda) = \lambda^2 \sum_{m=1}^{d} \frac{\varphi_m\,|\hat{v}_m|^2}{(\lambda-\varphi_m)^2} + z - \lambda = 0.$$

- Similarly, we can express objective value as $g(\lambda)$

Solve LQPQM

Optimality Conditions ($\nabla\mathcal{J}(\mathbf{q}) = 0$)

$$\nabla\mathcal{J}(\mathbf{q}) = \mathbf{q} - \frac{\mathbf{U}(\mathbf{q}-\mathbf{v})}{(\mathbf{q}+\mathbf{v})^H\mathbf{U}(\mathbf{q}+\mathbf{v})+z} = \mathbf{0} \Leftrightarrow \begin{cases} \mathbf{q} & = \frac{1}{\lambda}\mathbf{U}(\mathbf{q}+\mathbf{v}) \quad \text{(A1)} \\ \lambda & = (\mathbf{q}+\mathbf{v})^H\mathbf{U}(\mathbf{q}+\mathbf{v})+z \quad \text{(A2)} \end{cases}$$

Reduce to function of λ only

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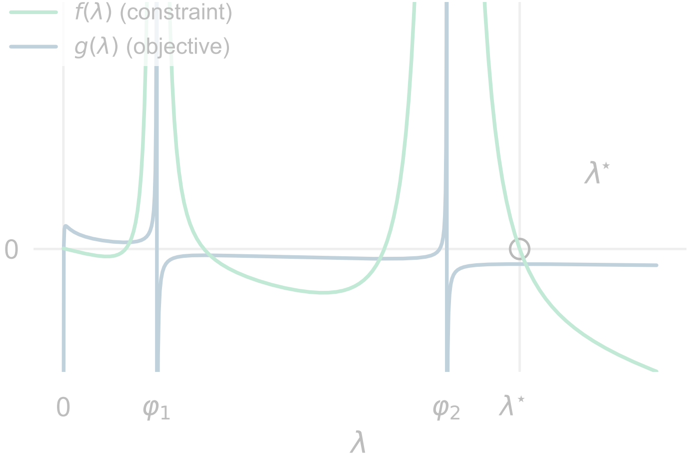
Reduce LQPQM to 1D Minimization

$$\min_{\lambda \in \mathbb{R}_+} g(\lambda) \quad \text{subject to} \quad f(\lambda) = 0$$

- Non-linear
- Local min. $f(\lambda) = 0$
- $g(\lambda_k) \geq g(\lambda_\ell), k \leq \ell$
- Use Newton-Raphson!



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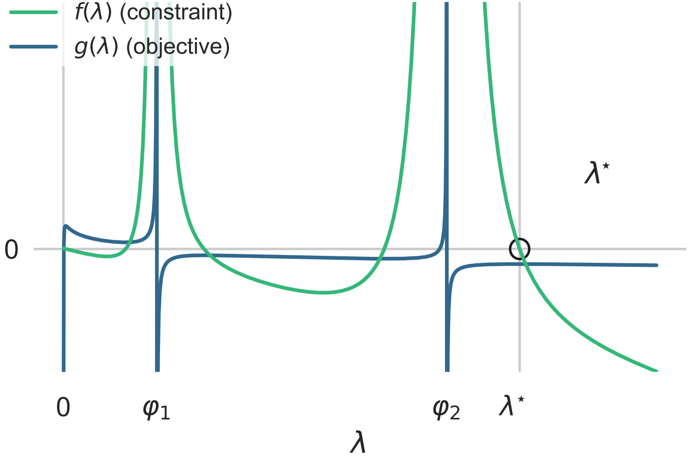

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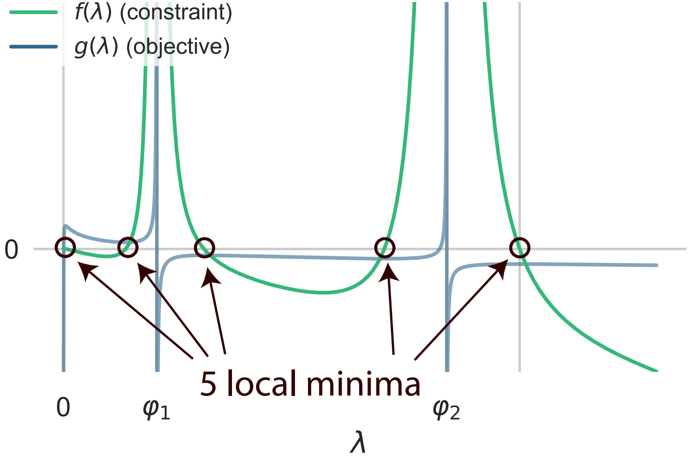

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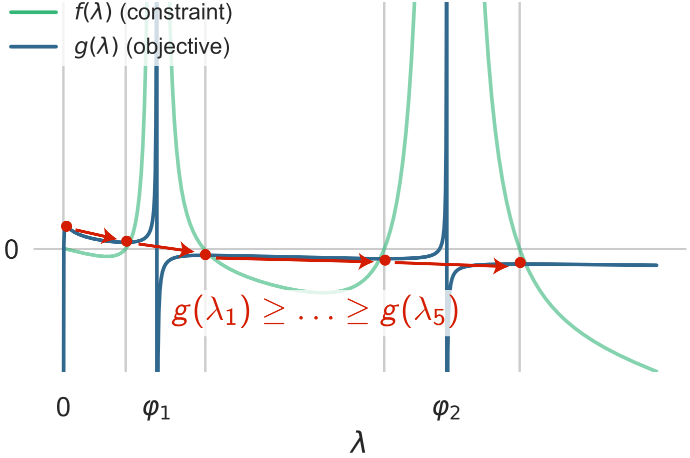

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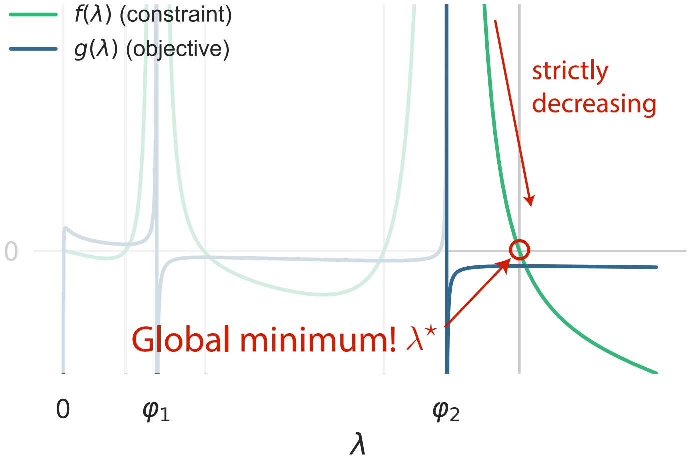

Experiment: Speed Contest

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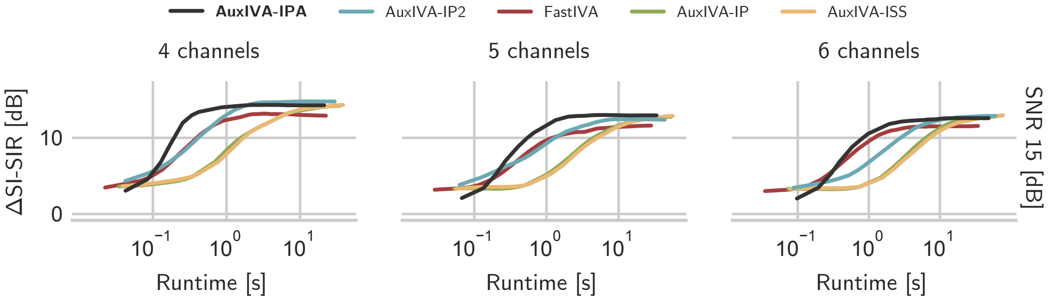
Convergence (SI-SIR) as function of runtime
(16 kHz, 1000 sim. rooms, SNR 15 dB)



New IPA Algorithm for BSS with AuxIVA

	IP [Ono2011]	**IP2** [Ono2018]	**ISS** [Scheibler2020]	**IPA**
Good separation	👍	👍	👍	👍
Cost per iteration	$O(M^3N)$	$O(M^3N)$	$O(M^2N)$	$O(M^3N)$
Inverse free	✗	✗	👍	✗
Speed	👍	👍👍	👍	👍👍👍

Exact Solution of LQPQM

$$\min_{\mathbf{q}\in\mathbb{C}^d} \mathbf{q}^H\mathbf{q} - \log((\mathbf{q}+\mathbf{v})^H\mathbf{U}(\mathbf{q}+\mathbf{v}) + z), \quad \mathbf{U} \in \mathrm{PSD}, \mathbf{v} \in \mathbb{C}^d, z \geq 0. \qquad \text{(LQPQM)}$$

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Conclusion

New IPA Algorithm for BSS with AuxIVA

	IP [Ono2011]	**IP2** [Ono2018]	**ISS** [Scheibler2020]	**IPA**
Good separation	👍	👍	👍	👍
Cost per iteration	$O(M^3N)$	$O(M^3N)$	$O(M^2N)$	$O(M^3N)$
Inverse free	✗	✗	👍	✗
Speed	👍	👍👍	👍	👍👍👍

Exact Solution of LQPQM

$$\min_{\mathbf{q}\in\mathbb{C}^d} \mathbf{q}^H\mathbf{q} - \log((\mathbf{q}+\mathbf{v})^H\mathbf{U}(\mathbf{q}+\mathbf{v})+z), \quad \mathbf{U}\in \mathrm{PSD}, \mathbf{v}\in\mathbb{C}^d, z\geq 0. \qquad \text{(LQPQM)}$$

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LINE Emojis

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